

3. Beispiel: Funktion 3. Grades, 3 Punkte + 1 Ableitungswert von $f'(x)$

I. $f(2)=5$; II. $f(0)=7$; III. $f(3)=0$; IV. $f'(2)=0$

Ansatz: $f(x) = ax^3 + bx^2 + cx + d$
 $f'(x) = 3ax^2 + 2bx + c$

I. $8a + 4b + 2c + d = 5$
 $8a + 4b + 2c + 7 = 5 \quad | -7$
 I. $8a + 4b + 2c = -2$
 III. $27a + 9b + 3c = -7$
 IV. $12a + 4b + c = 0$

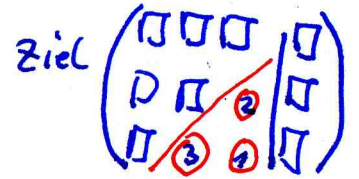
$$\left(\begin{array}{ccc|c} 8 & 4 & 2 & -2 \\ 27 & 9 & 3 & -7 \\ 12 & 4 & 1 & 0 \end{array} \right)$$

I. $a \cdot 2^3 + b \cdot 2^2 + c \cdot 2 + d = 5$
 $8a + 4b + 2c + d = 5$

II. $a \cdot 0 + b \cdot 0 + c \cdot 0 + d = 7 \Rightarrow d = 7$ schon
 gelöst, kommt
 nicht in
 die Matrix

III. $a \cdot 3^3 + b \cdot 3^2 + c \cdot 3 + 7 = 0 \quad | -7$
 $27a + 9b + 3c = -7$

IV. $3a \cdot 2^2 + 2b \cdot 2 + c = 0$
 $3a \cdot 4 + 2b \cdot 2 + c = 0$
 $12a + 4b + c = 0$



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Ziel $\left(\begin{array}{ccc|c} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \hline \square & \square & \square & \square \end{array} \right)$

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$$\left(\begin{array}{ccc|c} 8 & 4 & 2 & -2 \\ 27 & 9 & 3 & -7 \\ 12 & 4 & 1 & 0 \end{array} \right) \cdot 3$$

$$\left(\begin{array}{ccc|c} 8 & 4 & 2 & -2 \\ 27 & 9 & 3 & -7 \\ 36 & 12 & 3 & 0 \end{array} \right) \begin{array}{l} \left[\begin{array}{l} \leftarrow \\ \leftarrow \end{array} \right] \\ \leftarrow \end{array}$$

$$\left(\begin{array}{ccc|c} 8 & 4 & 2 & -2 \\ 27 & 9 & 3 & -7 \\ 9 & 3 & 0 & 7 \end{array} \right) \cdot 3 \quad \cdot 2 \quad \left(\begin{array}{ccc|c} 24 & 12 & 6 & -6 \\ -30 & -6 & 0 & 8 \\ 9 & 3 & 0 & 7 \end{array} \right) \cdot 2$$

$$\left(\begin{array}{ccc|c} 24 & 12 & 6 & -6 \\ 54 & 18 & 6 & -14 \\ 9 & 3 & 0 & 7 \end{array} \right) \begin{array}{l} \left[\begin{array}{l} \leftarrow \\ \leftarrow \end{array} \right] \\ \leftarrow \end{array} \quad \left(\begin{array}{ccc|c} 24 & 12 & 6 & -6 \\ -30 & -6 & 0 & 8 \\ 18 & 6 & 0 & 14 \end{array} \right) \begin{array}{l} \left[\begin{array}{l} \leftarrow \\ \leftarrow \end{array} \right] \\ \leftarrow \end{array}$$

a	b	c	
24	12	6	-6
-30	-6	0	8
-12	0	0	22

$$-12a = 22 \quad | : -12$$

$$a = \frac{22}{-12} = -\frac{11}{6}$$

$$-30 \cdot \frac{-11}{6} - 6b = 8$$

$$55 - 6b = 8 \quad | -55$$

$$-6b = -47 \quad | : (-6)$$

$$b = \frac{+47}{6}$$

$$24 \cdot \left(-\frac{11}{6}\right) + 12 \cdot \frac{47}{6} + 6c = -6$$

$$-44 + 94 + 6c = -6$$

$$50 + 6c = -6 \quad | -50$$

$$6c = -56 \quad | : 6$$

$$c = -\frac{28}{3}$$

weitere Beispiele

a) $f(-4) = 0,6$; $f(2) = 0,6$; $f(1) = -0,4$ hoch 2

b) $f(0) = -1$; $f(2) = 15$; $f(-1) = -15$; $f(3) = 41$ hoch 3

c) $f(3) = 11$; $f(0) = 2$; $f'(1) = 1$ hoch 2 + f'

d) $f(0) = -3$; $f(2) = -7,8$; $f(3) = -7,2$; $f'(1) = -2,8$ hoch 3 + f'

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d) $f(0) = -3; f(2) = -7,8; f(3) = -7,2; f'(1) = -2,8$ hoch 3 + f'

a) $16a - 4b + c = 0,6$

$$4a + 2b + c = 0,6$$

$$a + b + c = -0,4$$

$$\left(\begin{array}{ccc|c} 16 & -4 & 1 & 0,6 \\ 4 & 2 & 1 & 0,6 \\ 1 & 1 & 1 & -0,4 \end{array} \right)$$

b) $d = -1$

$$8a + 4b + 2c = 16$$

$$-a + b - c = -14$$

$$27a + 9b + 3c = 42$$

$$\left(\begin{array}{ccc|c} 8 & 4 & 2 & 16 \\ -1 & 1 & -1 & -14 \\ 27 & 9 & 3 & 42 \end{array} \right)$$

c) $9a + 3b + c = 11$

$$c = 2$$

$$2a + b = 1$$

$$\left(\begin{array}{cc|c} 9 & 3 & 9 \\ 2 & 1 & 1 \end{array} \right)$$

d) $d = -3$

$$8a + 4b + 2c = -4,8$$

$$27a + 9b + 3c = -4,2$$

$$3a + 2b + c = -2,8$$

$$\left(\begin{array}{ccc|c} 8 & 4 & 2 & -4,8 \\ 27 & 9 & 3 & -4,2 \\ 3 & 2 & 1 & -2,8 \end{array} \right)$$